A Lecture Series on DATA COMPRESSION

Subband Coding

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Motivation

- Problems with DCT-based compression
  - Blocking artifacts, especially at low bitrate
  - The methods for reducing blocking artifacts, such as overlapped transforms, are costly and complicated
  - Applying DCT on the whole image, rather than on small blocks, ignores the significant differences in frequency contents in various regions of the image, thus leading to less quality-bitrate performance

- Advantages of wavelets/subband coding
  - They operate on the whole image as one single block
  - Thus avoiding blocking artifacts
  - While dynamically adjusting the spatial/frequency resolution to the appropriate level in various regions of the image

- In practice, wavelets/subband coding performs as well as DCT and sometimes better, especially at low bitrate

Introduction
(Linear Filters)

- Definition of a linear filter
  - A linear filter f is characterized by a sequence $(f_k)_k$ of real numbers; the f_k's are called the *filter tabs*
  - Filtering an input signal $x = (x_n)_n$ through filter f gives an output signal $y = (y_n)_n$ where

$$y_n = \sum_k f_k x_{n-k} = \sum_k f_{n-k} x_k$$

  - That is, $y = f \otimes x$

- Let X, Y and F denote the Fourier Transforms of x, y and f, respectively

- Theorem: $Y = FX$

Linear Filters (Cont.)
Low-Pass Filters (LPF)

- An LPF eliminates the high-frequency contents of any input signal, and preserves the low-frequency contents

- An ideal LPF f must then have its Fourier Transform F as a nonzero constant in a frequency range $[0, a)$, and zero in the remaining range $[a, \pi]$

- Applications: Noise removal and image smoothing

Linear Filters (Cont.)
High-Pass Filters (HPF)

- High-pass filters (HPF)
- A HPF eliminates the low-frequency contents of any input signal, and preserves the high-frequency contents
- An ideal HPF f must then have its Fourier Transform F equal to zero in a frequency range $[0, a)$, and equal to a nonzero constant in the remaining range $[a, \pi]$
- Applications: Sharpening and edge detection

Linear Filters (Cont.)

- Ideal LPF's and HPF's are not realizable in practice, but many realizable filters are good approximations of ideal filters

- A filter is called a *finite-impulse-response* (FIR) filter if has a finite number of tabs; otherwise, the filter is called *an infinite-impulse response* (IIR) filter

Examples of LPF's and HPF's and their Effect

The Main Scheme of Subband Coding (Multirate Filter Banks)

How Subband Coding is Generally Applied

Issues

- Filter design
- Quantization Method
- Shape of the tree
- Same or different filter sets per image or class of images?

Filter Design

- Classical filter design techniques for LPF's and HPF's
  - Least Mean Square technique
  - Butterworth technique
  - Chebychev technique

- Those techniques are for designing single filters, rather than a bank of four filters working together

- The four filters for a subband coding system must have the **perfection reconstruction** property
  - the output signal is identical to the input signal if no quantization takes place

The Perfect Reconstruction Condition

- The z-transform of a sequence $(x_n)_n$ is $X(z) = \Sigma_n\, x_n z^n$

- If $(y_n)_n$ is the output of a linear filter $(f_k)_k$ given input $(x_n)_n$, then $Y(z) = F(z)P(z)$

- Therefore, for the subband coding scheme
  - $\hat{X}(z) = P(z)\hat{U}(z) + Q(z)\hat{V}(z)$
  - $\hat{U}(z) = \Sigma_n\, \hat{u}_n z^n = \Sigma_{n\ even}\, u_{\frac{n}{2}} z^n = \Sigma_n\, u_n z^{2n} = \Sigma_n\, \overline{u}_{2n} z^{2n}$
    $= \frac{1}{2}\left[\Sigma_n\, \overline{u}_n z^n + \Sigma_n\, \overline{u}_n z^{-n}\right]$
    $= \frac{1}{2}\left[\overline{U}(z) + \overline{U}(-z)\right] = \frac{1}{2}\left[G(z)X(z) + G(-z)X(-z)\right]$
  - Similarly, $\hat{V}(z) = \frac{1}{2}\left[H(z)X(z) + H(-z)X(-z)\right]$
  - Thus, $\hat{X}(z) = \frac{1}{2}P(z)\left[G(z)X(z) + G(-z)X(-z)\right] + \frac{1}{2}Q(z)\left[H(z)X(z) + H(-z)X(-z)\right]$
  - By regrouping we get
    $$\hat{X}(z) = \frac{1}{2}\left[G(z)P(z) + H(z)Q(z)\right]X(z) + \frac{1}{2}\left[G(-z)P(z) + H(-z)Q(z)\right]X(-z)$$

- To have $\hat{x} = x$, we must have $\hat{X}(z) = X(z)$, leading to the following perfect reconstruction (PR) condition:

$$G(z)P(z) + H(z)Q(z) = 2$$
$$G(-z)P(z) + H(-z)Q(z) = 0$$

- Consequently, to get a subband filter bank (of four filters), one has to solve the two equations above, subject to the constraints that
  - For HPF's
    * $H(1) = Q(1) = 0$
    * $H(-1) \neq 0$
    * $Q(-1) \neq 0$
  - For LPF's
    * $G(-1) = P(-1) = 0$
    * $H(1) \neq 0$
    * $Q(1) \neq 0$

Examples of Good and Bad Filters that Satisfy PR

Quantization

- Quantization approaches of the subbands
  - Uniform scalar quantization
  - Non-uniform scalar quantization
  - Vector quantization
  - One quantizer for all the subbands, or
  - Different quantizers for different subbands

- Prospects for optimal Max-LLoyd scalar quantization of high-frequency subbands
  - Probability distribution of the pixel values in HF subbands: The generalized Gaussian distribution

    $$p(x) = ae^{-|bx|^r}$$

    where

    $$b = \frac{1}{\sigma}\left(\frac{\Gamma(\frac{3}{r})}{\Gamma(\frac{1}{r})}\right)^{\frac{1}{2}}$$
    $$a = \frac{br}{2\Gamma(\frac{1}{r})}$$

    and σ is the standard deviation of the underlying data

  - Experimentation has shown that r is about 0.7

- Therefore, the sender need not send the decision levels and reconstruction levels of the Max-Lloyd quantizer; rather, only the standard deviation σ need be sent.

- Question: Can the decision levels and quantization levels of the Max-Lloyd quantizer be determined ANALYTICALLY for the case of GG probability distribution?

Quantization (Cont.)
(Vector Quantization of Subbands)

- Is VQ needed for high-frequency subbands?
  - Answer: It depends of how good the filters are
  - Under ideal filters, the high frequency coefficients are completely decorrelated, making VQ unnecessary (and rather undesirable)
  - In practice, the farther the filters are from ideal, the more correlation "leaks" into the high-frequency subbands, thus opening the door for VQ
  - With the commonly used filters, there is some correlation leakage; but there is still the tradeoff between the slight improvement brought by VQ and the high time overhead associated with VQ

- Design Issues for VQ in subband coding
  - One VQ table for all subbands, or
  - One VQ table per subband, or
  - One VQ table for the subbands of a whole class of images?
  - How large should the vector size be?

Shape of the Decomposition Tree

- Mallat Shape

- FBI Shape

Shape of the Decomposition Tree (Cont.) (Research Issues)

- What is the best shape?
- Is there a best shape for all images, or at least one best shape per class of images?
- If not, is there an efficient way of deciding the shape of the tree on-line?

Same or Different Filters for Different Subbands?

- Intuitively, the best filter set for a given signal is the one whose corresponding wavelet best resembles the signal in shape (i.e., in plot)

- The data in the subbands have different plots than the original data, suggesting the use for different filters than the ones applied on the original data

- For better understanding of this issue, one has to draw on the insight provided by wavelet theory, which is the subject matter of next lecture